



SAMPLE MATERIAL



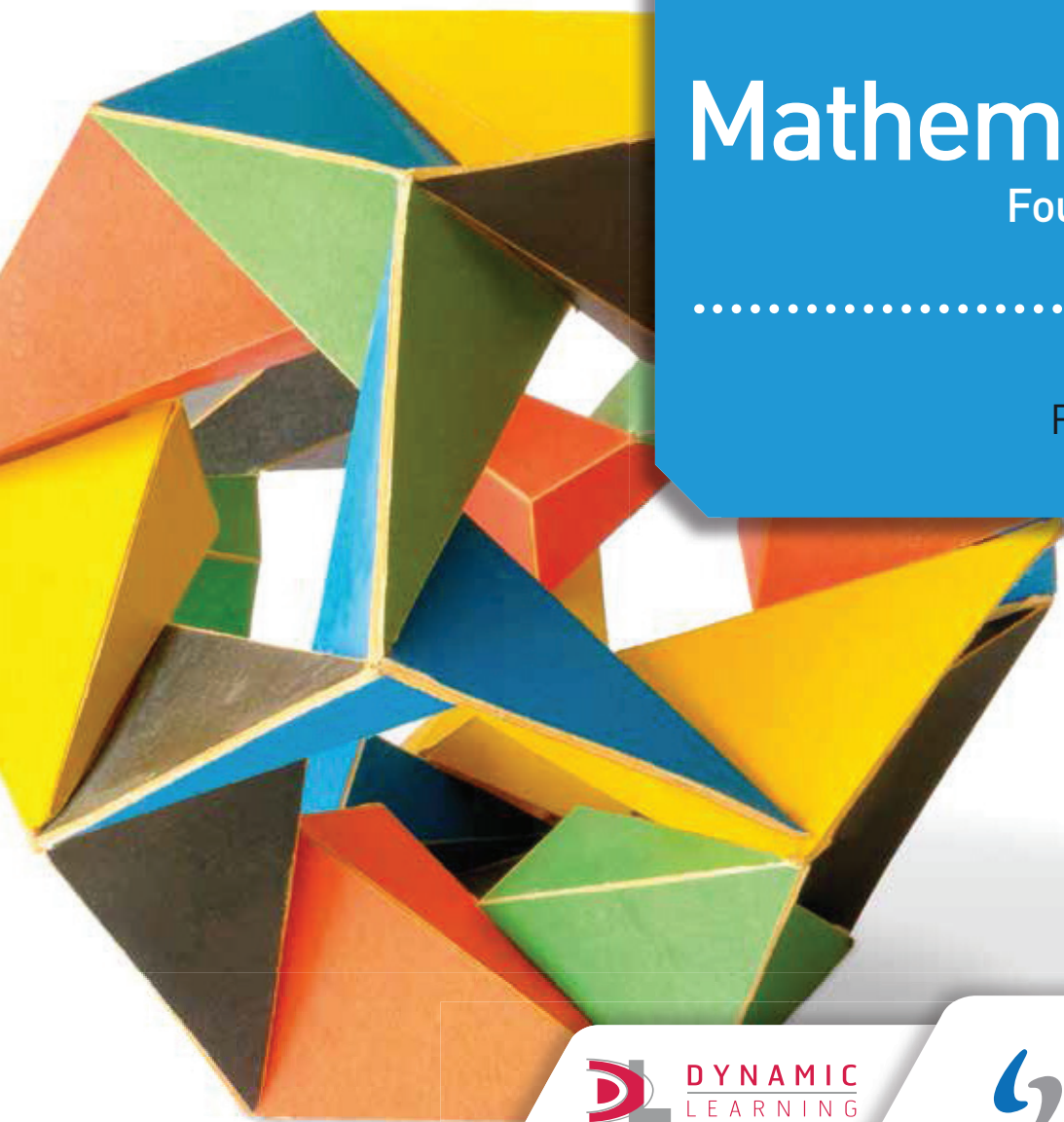
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Cambridge IGCSE®

Core
Mathematics
Fourth edition

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Terry Wall



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Cambridge IGCSE® Core Mathematics	ISBN 9781510421660	March 2018
Cambridge IGCSE® Core Mathematics Workbook	ISBN 9781510421677	June 2018
Cambridge IGCSE® Core Mathematics Student eTextbook	ISBN 9781510420595	April 2018
Cambridge IGCSE® Core Mathematics Whiteboard eTextbook	ISBN 9781510420601	March 2018

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TOPIC 2

Algebra and graphs

Contents

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Topic 2 Mathematical investigations and ICT

Syllabus

C2.1

Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in formulae. Transform simple formulae. Construct simple expressions and set up simple equations.

C2.2

Manipulate directed numbers. Use brackets and extract common factors. Expand products of algebraic expressions.

C2.4

Use and interpret positive, negative and zero indices. Use the rules of indices.

C2.5

Derive and solve simple linear equations in one unknown. Derive and solve simultaneous linear equations in two unknowns.

C2.7

Continue a given number sequence. Recognise patterns in sequences including the term to term rule and relationships between different sequences. Find and use the n th term of sequences.

C2.10

Interpret and use graphs in practical situations including travel graphs and conversion graphs. Draw graphs from given data.

C2.11

Construct tables of values for functions of the form $ax + b$, $\pm x^2 + ax + b$, $a - x$ ($x \neq 0$) where a and b are integer constants. Draw and interpret these graphs. Solve linear and quadratic equations approximately, including finding and interpreting roots by graphical methods. Recognise, sketch and interpret graphs of functions.

The development of algebra

The roots of algebra can be traced to the ancient Babylonians, who used formulae for solving problems. However, the word *algebra* comes from the Arabic language. Muḥammad ibn Mūsā al-Khwārizmī (AD790–850) wrote *Kitāb al-Jabr* (*The Compendious Book on Calculation by Completion and Balancing*), which established algebra as a mathematical subject. He is known as the father of algebra.

Persian mathematician Omar Khayyam (AD1048–1131), who studied in Bukhara (now in Uzbekistan), discovered algebraic geometry and found the general solution of a cubic equation.

In 1545, Italian mathematician Girolamo Cardano published *Ars Magna* (*The Great Art*), a forty-chapter book in which he gave, for the first time, a method for solving a quartic equation.

Graphs of functions

Linear functions

A linear function produces a straight line when plotted. A straight line consists of an infinite number of points. However, to plot a linear function, only two points on the line are needed. Once these have been plotted, the line can be drawn through them and extended, if necessary, at both ends.

→ Worked examples

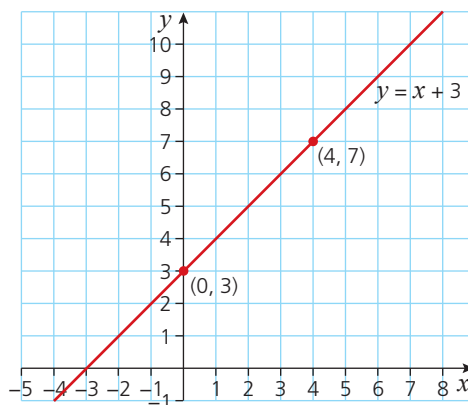
1 Plot the line $y = x + 3$.

Solution

To identify two points simply choose two values of x , substitute these into the equation and calculate the corresponding y -values. Sometimes a small table of results is clearer.

x	y
0	3
4	7

Using the table, two points on the line are $(0, 3)$ and $(4, 7)$. Plot the points on a pair of axes and draw a line through them:



It is good practice to check with a third point:

Substituting $x = 2$ into the equation gives $y = 5$. As the point $(2, 5)$ lies on the line, the line is drawn correctly.

2 Plot the line $2y = x + 6$.

Solution

It is often easier to plot a line if the function is first written with y as the subject:

$$2y + x = 6$$

$$2y = -x + 6$$

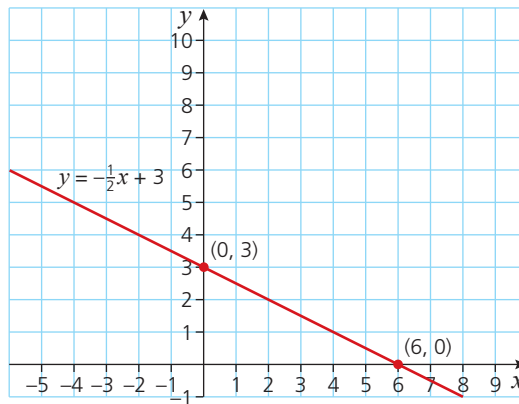
$$y = -\frac{1}{2}x + 3$$

Choose two values of x and find the corresponding values of y :

x	y
0	3
6	0

From the table, two points on the line are $(0, 3)$ and $(6, 0)$.

Plot the points on a pair of axes and draw a line through them:



Check with a third point:

Substituting $x = 4$ into the equation gives $y = 1$. As the point $(4, 1)$ lies on the line, the line is drawn correctly.

Exercise 16.1

1 Plot the following straight lines.

a $y = 2x + 4$

b $y = 2x + 3$

c $y = 2x - 1$

d $y = x - 4$

e $y = x + 1$

f $y = x + 3$

g $y = 1 - x$

h $y = 3 - x$

i $y = -(x + 2)$

2 Plot the following straight lines.

a $y = 2x + 3$

b $y = x - 4$

c $y = 3x - 2$

d $y = -2x$

e $y = -x - 1$

f $-y = x + 1$

g $-y = 3x - 3$

h $2y = 4x - 2$

i $y - 4 = 3x$

3 Plot the following straight lines.

a $-2x + y = 4$

b $-4x + 2y = 12$

c $3y = 6x - 3$

d $2x = x + 1$

e $3y - 6x = 9$

f $2y + x = 8$

g $x + y + 2 = 0$

h $3x + 2y - 4 = 0$

i $4 = 4y - 2x$

Graphical solution of simultaneous equations

When solving two equations simultaneously, you need to find a solution that works for both equations. Chapter 13 shows how to arrive at the solution algebraically. It is, however, possible to arrive at the same solution graphically.

→ Worked examples

- a** By plotting the graphs of both of the following equations on the same axes, find a common solution.

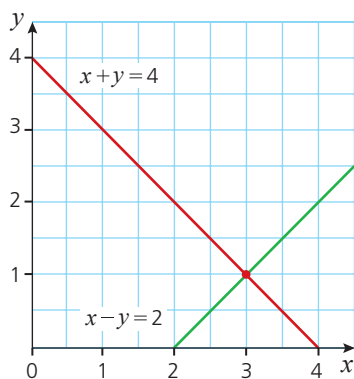
$$x + y = 4$$

$$x - y = 2$$

- b** Check your answer to part **a** by solving the equations algebraically.

Solution

- a** When both lines are plotted, the point at which they cross gives the common solution. This is because it is the only point which lies on both lines.



Therefore the common solution is (3, 1).

b $x + y = 4$ (1)

$x - y = 2$ (2)

(1) + (2) $\rightarrow 2x = 6$

$x = 3$

Substituting $x = 3$ into equation (1):

$3 + y = 4$

$y = 1$

Therefore the common solution occurs at (3, 1).

Exercise 16.2

Solve the simultaneous equations below:

i) by graphical means

ii) by algebraic means.

- | | | |
|---|---|---|
| 1 a $x + y = 5$
$x - y = 1$ | b $x + y = 7$
$x - y = 3$ | c $2x + y = 5$
$x - y = 1$ |
| d $2x + 2y = 6$
$2x - y = 3$ | e $x + 3y = -1$
$x - 2y = -6$ | f $x - y = 6$
$x + y = 2$ |
| 2 a $3x - 2y = 13$
$2x + y = 4$ | b $4x - 5y = 1$
$2x + y = -3$ | c $x + 5 = y$
$2x + 3y - 5 = 0$ |
| d $x = y$
$x + y + 6 = 0$ | e $2x + y = 4$
$4x + 2y = 8$ | f $y - 3x = 1$
$y = 3x - 3$ |

Quadratic functions

The general expression for a quadratic function takes the form $ax^2 + bx + c$, where a , b and c are constants. Some examples of quadratic functions are:

$$y = 2x^2 + 3x - 12 \quad y = x^2 - 5x + 6 \quad y = 3x^2 + 2x - 3$$

If a graph of a quadratic function is plotted, the smooth curve produced is called a **parabola**.

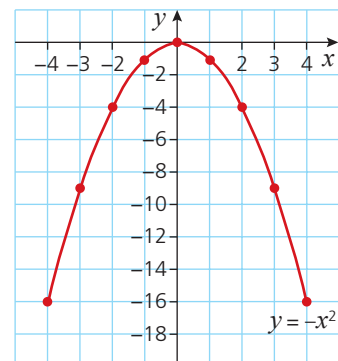
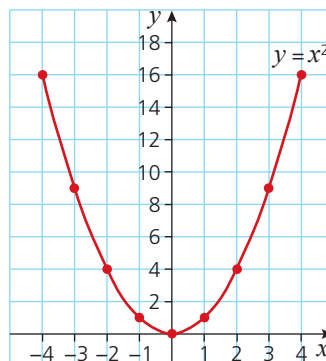
For example:

$$y = x^2$$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

$$y = -x^2$$

x	-4	-3	-2	-1	0	1	2	3	4
y	-16	-9	-4	-1	0	-1	-4	-9	-16



→ Worked examples

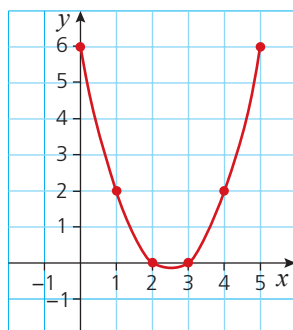
- 1 Plot a graph of the function $y = x^2 - 5x + 6$ for $0 \leq x \leq 5$.

Solution

First create a table of values for x and y :

x	0	1	2	3	4	5
$y = x^2 - 5x + 6$	6	2	0	0	2	6

These can then be plotted to give the graph:



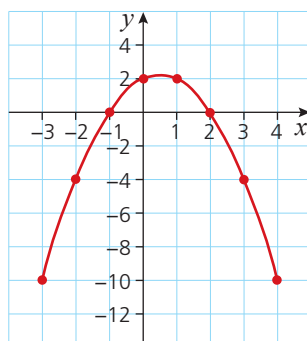
- 2 Plot a graph of the function $y = -x^2 + x + 2$ for $-3 \leq x \leq 4$.

Solution

Draw up a table of values:

x	-3	-2	-1	0	1	2	3	4
$y = -x^2 + x + 2$	-10	-4	0	2	2	0	-4	-10

Then plot the points and join them with a smooth curve:



Exercise 16.3

For each of the following quadratic functions, construct a table of values for the stated range and then draw the graph.

- 1 $y = x^2 + x - 2$, $-4 \leq x \leq 3$
- 2 $y = -x^2 + 2x + 3$, $-3 \leq x \leq 5$
- 3 $y = x^2 - 4x + 4$, $-1 \leq x \leq 5$
- 4 $y = -x^2 - 2x - 1$, $-4 \leq x \leq 2$
- 5 $y = x^2 - 2x - 15$, $-4 \leq x \leq 6$

Graphical solution of a quadratic equation

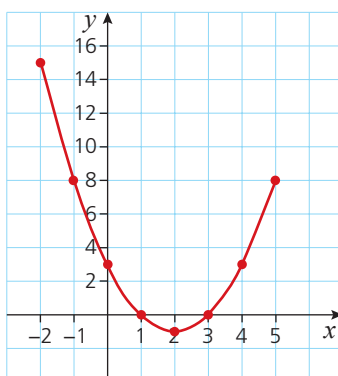
To solve an equation, you need to find the values of x when $y = 0$. On a graph, these are the values of x where the curve crosses the x -axis. These are known as the **roots** of the equation.

→ Worked examples

- 1 Draw a graph of $y = x^2 - 4x + 3$ for $-2 \leq x \leq 5$.
- 2 Use the graph to solve the equation $x^2 - 4x + 3 = 0$.

Solution

x	-2	-1	0	1	2	3	4	5
y	15	8	3	0	-1	0	3	8



These are the values of x where the graph crosses the x -axis.

- 2 The solutions are $x = 1$ and $x = 3$.

Exercise 16.4

Find the roots of each of the quadratic equations below by plotting a graph for the ranges of x stated.

- 1 $x^2 - x - 6 = 0$, $-4 \leq x \leq 4$
- 2 $-x^2 + 1 = 0$, $-4 \leq x \leq 4$
- 3 $x^2 - 6x + 9 = 0$, $0 \leq x \leq 6$
- 4 $-x^2 - x + 12 = 0$, $-5 \leq x \leq 4$
- 5 $x^2 - 4x + 4 = 0$, $-2 \leq x \leq 6$

In the previous worked example, $y = x^2 - 4x + 3$, a solution was found to the equation $x^2 - 4x + 3 = 0$ by reading the values of x where the graph crossed the x -axis. The graph can, however, also be used to solve other related quadratic equations.

→ Worked example

Use the graph of $y = x^2 - 4x + 3$ to solve the equation $x^2 - 4x + 1 = 0$.

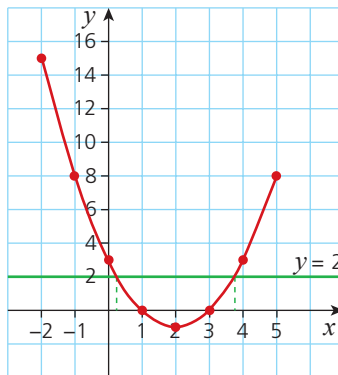
Look at how the given equation relates to the given graph.

Solution

$x^2 - 4x + 1 = 0$ can be rearranged to give:

$$x^2 - 4x + 3 = 2$$

Using the graph of $y = x^2 - 4x + 3$ and plotting the line $y = 2$ on the same axes gives the graph shown below:



Where the curve and the line cross gives the solution to $x^2 - 4x + 3 = 2$, and hence also $x^2 - 4x + 1 = 0$.

Therefore the solutions to $x^2 - 4x + 1 = 0$ are $x \approx 0.3$ and $x \approx 3.7$.

Exercise 16.5

Using the graphs that you drew in Exercise 16.4, solve the following quadratic equations. Show your method clearly.

- 1 $x^2 - x - 4 = 0$
- 2 $-x^2 - 1 = 0$
- 3 $x^2 - 6x + 8 = 0$
- 4 $-x^2 - x + 9 = 0$
- 5 $x^2 - 4x + 1 = 0$

The reciprocal function

If a graph of a reciprocal function is plotted, the curve produced is called a **hyperbola**.

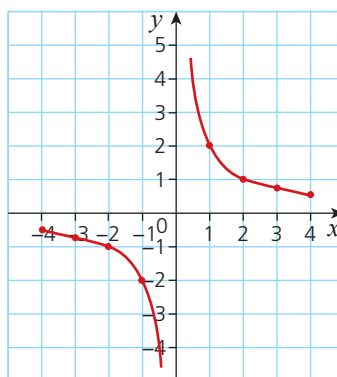
→ Worked examples

Draw the graph of $y = \frac{2}{x}$ for $-4 \leq x \leq 4$.

Solution

x	-4	-3	-2	-1	0	1	2	3	4
y	-0.5	-0.7	-1	-2	-	2	1	0.7	0.5

$y = \frac{2}{x}$ is a reciprocal function so the graph is a hyperbola:



Exercise 16.6

- 1 Plot the graph of the function $y = \frac{1}{x}$ for $-4 \leq x \leq 4$.
- 2 Plot the graph of the function $y = \frac{3}{x}$ for $-4 \leq x \leq 4$.
- 3 Plot the graph of the function $y = \frac{5}{2x}$ for $-4 \leq x \leq 4$.

Recognising and sketching functions

So far in this chapter, you have plotted graphs of functions. In other words, you have substituted values of x into the equation of a function, calculated the corresponding values of y , and plotted and joined the resulting (x, y) coordinates.

However, plotting an accurate graph is time-consuming and is not always necessary to answer a question. In many cases, a sketch of a graph is as useful and is considerably quicker.

When doing a sketch, certain key pieces of information need to be included. As a minimum, where the graph intersects both the x -axis and y -axis needs to be given.

Sketching linear functions

Straight line graphs can be sketched simply by working out where the line intersects both axes.

→ Worked example

Sketch the graph of $y = -3x + 5$.

Solution

The graph intersects the y -axis when $x = 0$.

This is substituted into the equation:

$$y = -3(0) + 5.$$

$$y = 5$$

The graph intersects the x -axis when $y = 0$.

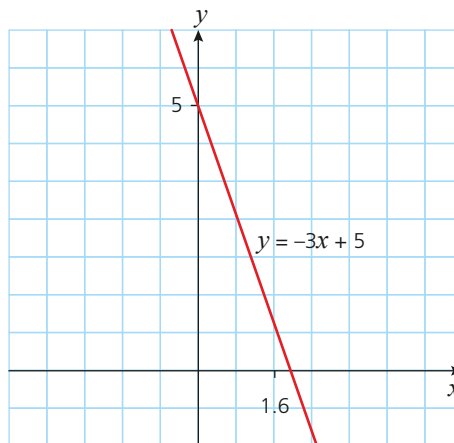
This is then substituted into the equation and solved:

$$0 = -3x + 5$$

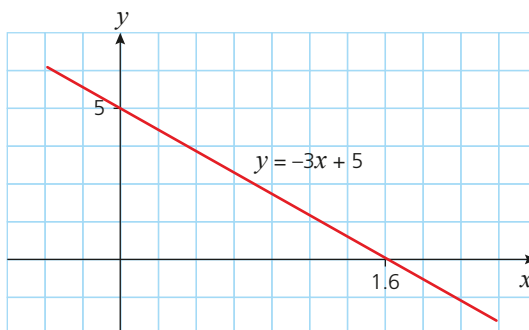
$$3x = 5$$

$$x = \frac{5}{3} \text{ (or } 1.\dot{6})$$

Mark the two points and join them with a straight line:



Note that the sketch below, although it looks very different to the one above, is also acceptable as it shows the same intersections with the axes.



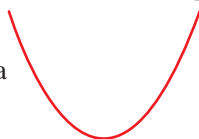

Exercise 16.7

Sketch the following linear functions, showing clearly where the lines intersect both axes.

- 1 $y = 2x - 4$
- 2 $y = \frac{1}{2}x + 6$
- 3 $y = -2x - 3$
- 4 $y = -\frac{1}{3}x + 9$
- 5 $2y + x - 2 = 0$
- 6 $x = \frac{2y + 4}{3}$

Sketching quadratic functions

You will have seen that plotting a quadratic function produces

either a  or a  shaped curve,

depending on whether the x^2 term is positive or negative.

Therefore, a quadratic graph has either a highest point or a lowest point, depending on its shape. These points are known as **turning points**.

The key points to include in a sketch of a quadratic graph are the intersection with the y -axis, and either the coordinates of the turning point or the intersection(s) with the x -axis.

To sketch a quadratic, the key points that need to be included are the intersection with the y -axis and either the coordinates of the turning point or the intersection(s) with the x -axis.

→ Worked examples

- 1 The graph of the quadratic equation $y = x^2 - 6x + 11$ has a turning point at $(3, 2)$. Sketch the graph, showing clearly where it intersects the y -axis.

Solution

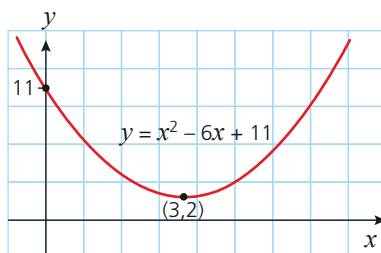
As x^2 term is positive, the graph is \cup -shaped.

To find where the graph intersects the y -axis, substitute $x = 0$ into the equation.

$$y = (0)^2 - 6(0) + 11$$

$$y = 11$$

Therefore the graph can be sketched as follows:



- 2 The coordinates of the turning point of a quadratic graph are $(-4, 4)$. The equation of the function is $y = -x^2 - 8x - 12$. Sketch the graph.

Solution

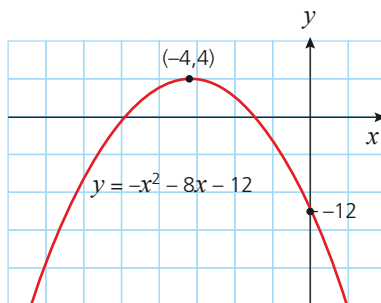
As the x^2 term is negative, the graph is \cap -shaped.

To find where it intersects the y -axis, substitute $x = 0$ into the equation.

$$y = -(0)^2 - 8(0) - 12$$

$$y = -12$$

Therefore the graph of $y = -x^2 - 8x - 12$ can be sketched as follows:



You will have seen in Chapter 11, that an expression such as $(x - 2)(x + 3)$ can be expanded to give $x^2 + x - 6$. Therefore $(x - 2)(x + 3)$ is also a quadratic expression. If a quadratic function is given in this form then the graph can still be sketched and, in particular, the points where it crosses the x -axis can be found too.

→ Worked example

- 1 Show that $(x + 4)(x + 2) = x^2 + 6x + 8$.
- 2 Sketch the graph of $y = (x + 4)(x + 2)$.

Solution

$$\begin{aligned} 1 \quad (x + 4)(x + 2) &= x^2 + 4x + 2x + 8 \\ &= x^2 + 6x + 8 \end{aligned}$$

- 2 The x^2 term is positive so the graph is \cup -shaped.
The graph intersects the y -axis when $x = 0$:

$$\begin{aligned} y &= (0 + 4)(0 + 2) \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

As the coordinates of the turning point are not given, the intersection with the x -axis needs to be calculated. At the x -axis, $y = 0$.
Substituting $y = 0$ into the equation gives:

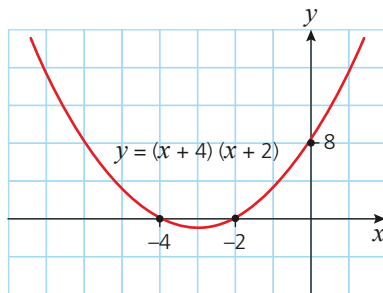
$$(x + 4)(x + 2) = 0$$

For the product of two terms to be zero, one of the terms must be zero.

$$\text{If } (x + 4) = 0, \text{ then } x = -4.$$

$$\text{If } (x + 2) = 0, \text{ then } x = -2.$$

$x = -4$ and $x = -2$ are therefore where the graph intersects the x -axis.
The graph can be sketched as:



Exercise 16.8

- 1 For each part, the equation of a quadratic function and the coordinates of its turning point are given. Sketch a graph for each function.

	Quadratic function	Turning point
a	$y = x^2 - 10x + 27$	$(5, 2)$
b	$y = x^2 + 2x - 5$	$(-1, -6)$
c	$y = -x^2 + 4x - 3$	$(2, 1)$
d	$y = x^2 - 12x + 36$	$(6, 0)$
e	$y = 4x^2 - 20x$	$(\frac{5}{2}, -25)$

- 2 a Expand the brackets $(x + 3)(x - 3)$.
b Use your expansion in part a) to sketch the graph of $y = (x + 3)(x - 3)$. Label any point(s) of intersection with the axes.
- 3 a Expand the expression $-(x - 2)(x - 4)$.
b Use your expansion in part a) to sketch the graph of $y = -(x - 2)(x - 4)$. Label any point(s) of intersection with the axes.
- 4 a Expand the brackets $(-3x - 6)(x + 1)$.
b Use your expansion in part a) to sketch the graph of $y = (-3x - 6)(x + 1)$. Label any point(s) of intersection with the axes.



Student assessment 1

- 1 Plot these lines on the same pair of axes. Label each line clearly.
a $x = -2$ b $y = 3$ c $y = 2x$ d $y = -\frac{x}{2}$
- 2 Plot the graph of each linear equation.
a $y = x + 1$ b $y = 3 - 3x$
c $2x - y = 4$ d $2y - 5x = 8$
- 3 Solve each pair of simultaneous equations graphically.
a $x + y = 4$ b $3x + y = 2$
 $x - y = 0$ $x - y = 2$
c $y + 4x + 4 = 0$ d $x - y = -2$
 $x + y = 2$ $3x + 2y + 6 = 0$
- 4 a Copy and complete the table of values for the function $y = x^2 + 8x + 15$.
- | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|---|---|---|
| x | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| y | | -3 | | | | -3 | | | | |
- b Plot a graph of the function.
- 5 Plot the graph of each function for the given limits of x .
a $y = x^2 - 3$; $-4 \leq x \leq 4$ c $y = -x^2 - 2x - 1$; $-3 \leq x \leq 3$
b $y = 3 - x^2$; $-4 \leq x \leq 4$ d $y = x^2 + 2x - 7$; $-5 \leq x \leq 3$
- 6 a Plot the graph of the quadratic function $y = x^2 + 9x + 20$ for $-7 \leq x \leq -2$.
b Showing your method clearly, use information from your graph to solve the equation $x^2 = -9x - 14$.
- 7 Plot the graph of $y = \frac{1}{x}$ for $-4 \leq x \leq 4$.
- 8 Sketch the graph of $y = \frac{1}{3}x - 5$. Label clearly any points of intersection with the axes.
- 9 The quadratic equation $y = 4x^2 - 8x - 4$ has its turning point at $(1, -8)$. Sketch the graph of the function.

? Student assessment 2

- 1 Plot these lines on the same pair of axes. Label each line clearly.
a $x = 3$ **b** $y = -2$ **c** $y = -3x$ **d** $y = \frac{x}{4} + 4$
- 2 Plot the graph of each linear equation.
a $y = 2x + 3$ **b** $y = 4 - x$
c $2x - y = 3$ **d** $-3x + 2y = 5$
- 3 Solve each pair of simultaneous equations graphically.
a $x + y = 6$ **b** $x + 2y = 8$
 $x - y = 0$ $x - y = -1$
c $2x - y = -5$ **d** $4x - 2y = -2$
 $x - 3y = 0$ $3x - y + 2 = 0$
- 4 **a** Copy and complete the table of values for the function $y = -x^2 - 7x - 12$.

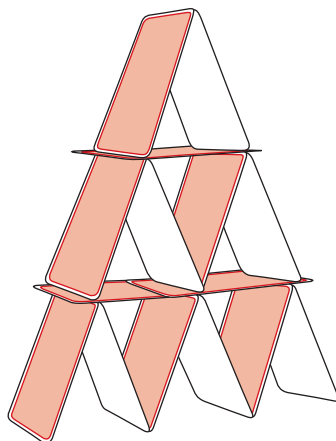
x	-7	-6	-5	-4	-3	-2	-1	0	1	2
y		-6				-2				

b Plot a graph of the function.
- 5 Plot the graph of each function for the given limits of x .
a $y = x^2 - 5$; $-4 \leq x \leq 4$ **b** $y = 1 - x^2$; $-4 \leq x \leq 4$
c $y = x^2 - 3x - 10$; $-3 \leq x \leq 6$ **d** $y = -x^2 - 4x - 4$; $-5 \leq x \leq 1$
- 6 **a** Plot the graph of the quadratic equation $y = -x^2 - x + 15$ for $-6 \leq x \leq 4$.
b Showing your method clearly, use your graph to help you solve these equations:
i) $10 = x^2 + x$ **ii)** $x^2 = -x + 5$
- 7 Plot the graph of $y = \frac{2}{x}$ for $-4 \leq x \leq 4$
- 8 Sketch the graph of $y = -\frac{5}{2}x + 10$. Label clearly any points of intersection with the axes.
- 9 Sketch the graph of the quadratic equation $y = -(2x - 2)(x - 7)$. Show clearly any points of intersection with the axes.

Mathematical investigations and ICT

House of cards

The drawing shows a house of cards 3 layers high. 15 cards are needed to construct it.

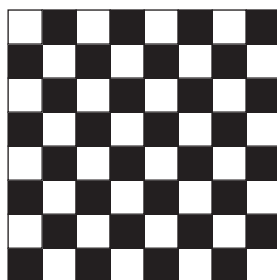


- 1 How many cards are needed to construct a house 10 layers high?
- 2 The largest house of cards ever constructed was 75 layers high. How many cards were needed?
- 3 Show that the general formula for the number of cards, c , needed to construct a house of cards n layers high is:

$$c = \frac{1}{2}n(3n + 1)$$

Chequered boards

A chessboard is an 8×8 square grid with alternating black and white squares as shown:



There are 64 unit squares of which 32 are black and 32 are white.

Consider boards of different sizes consisting of alternating black and white unit squares.

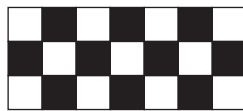
For example:



Total number of unit squares: 30

Number of black squares: 15

Number of white squares: 15



Total number of unit squares: 21

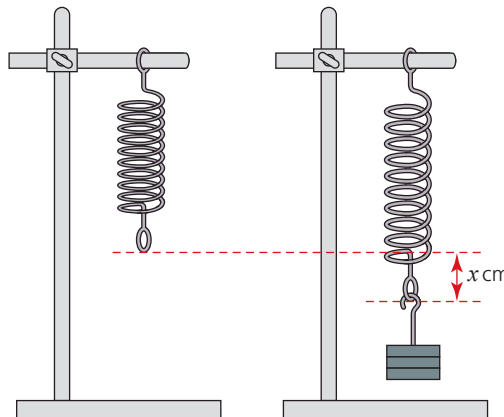
Number of black squares: 10

Number of white squares: 11

- 1 Investigate the number of black and white unit squares on different rectangular boards. Note: For consistency you may find it helpful to always keep the bottom right-hand square the same colour.
- 2 What are the numbers of black and white squares on a board $m \times n$ units?

Modelling: stretching a spring

A spring is attached to a clamp stand as shown below:



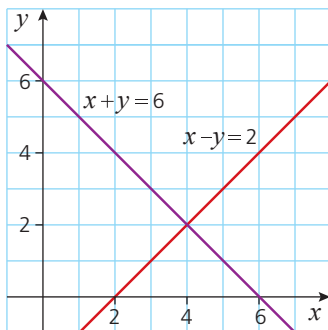
Different weights are attached to the end of the spring.

The table shows the mass (m) in grams of each weight and the amount by which the spring stretches (x) in centimetres.

Mass (g)	50	100	150	200	250	300	350	400	450	500
Extension (cm)	3.1	6.3	9.5	12.8	15.4	18.9	21.7	25.0	28.2	31.2

- 1 Plot a graph of mass against extension.
- 2 Describe the approximate relationship between the mass and the extension.
- 3 Draw a line of best fit through the data.
- 4 Calculate the equation of the line of best fit.
- 5 Use your equation to predict the extension of the spring for a mass of 275 g.
- 6 Explain why it is unlikely that the equation would be useful to find the extension when a mass of 5 kg is added to the spring.

ICT activity 1



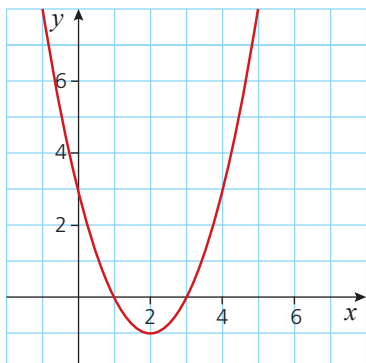
You have seen that the solution of two simultaneous equations gives the coordinates of the point that satisfies both equations. If the simultaneous equations were plotted, the point at which the lines cross would correspond to the solution of the simultaneous equations.

For example:

Solving $x + y = 6$ and $x - y = 2$ produces the result $x = 4$ and $y = 2$, i.e. coordinates (4, 2).

Plotting the graphs of both lines, as shown on the left, confirms that the point of intersection occurs at (4, 2).

- 1 Use a graphing package to find the simultaneous solution of each pair of equations.
 - a $y = x$ and $x + y = 4$
 - b $y = 2x$ and $x + y = 3$
 - c $y = 2x$ and $y = 3$
 - d $y - x = 2$ and $y + \frac{1}{2}x = 5$
 - e $y + x = 5$ and $y + \frac{1}{2}x = 5$
- 2 Check your answers to Q1 by solving each pair of simultaneous equations algebraically.



ICT activity 2

In this activity you will be using a graphing package or graphical calculator to find the solutions to quadratic and reciprocal functions.

You have seen that if a quadratic equation is plotted, its solution is represented by the points of intersection with the x -axis. For example, when plotting $y = x^2 - 4x + 3$, as shown on the left, the solution of $x^2 - 4x + 3 = 0$ occurs where the graph crosses the x -axis, i.e. at $x = 1$ and $x = 3$.

Use a graphing package or a graphical calculator to solve each equation graphically.

1 $x^2 + x - 2 = 0$

2 $x^2 - 7x + 6 = 0$

3 $x^2 + x - 12 = 0$

4 $2x^2 + 5x - 3 = 0$

5 $\frac{2}{x} - 2 = 0$

6 $\frac{2}{x} + 1 = 0$

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